Public Investment, Health Infrastructure and Income Growth

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Abstract

Based on individual occupational choice in a model including a production function with public investment and public health infrastructure, this paper presents an examination of how allocation of public investment and public health infrastructure affects the dynamics of income. Individuals work as skilled laborers or unskilled laborers, as in the model described by Caselli (1999), and educational costs are necessary to work as a skilled laborer. Results show that government should provide both public investment and public health infrastructure to escape from the poverty trap with low income. Moreover, based on an initial allocation between public investment and public health infrastructure, it is decided how the government should form a policy to increase income growth.

JEL Classification: H54, E20

Key Words: Public investment, Health infrastructure, Life expectancy, Occupational choice, Economic growth

1 Introduction

As explained by Foster and Briceño-Garmendia (2010) and UN-HABITAT (2011), African countries must be provided infrastructure (Transport, Modern Energy, Telecoms, Water System, Sanitation, and so on) to foster economic growth and to escape from poverty. Providing infrastructure can achieve the Millennium Development Goals. UN-HABITAT (2011) introduces to the macroeconomic empirical literature the idea that the development of infrastructure brings about economic growth and productivity effects (Estache, Speciale and Veredas (2005), Ayogu (2007)). In developed countries, that infrastructure is sufficiently provided, but not in developing countries, especially in Sub-Saharan African countries.[1] Therefore, the government must carry out policies to increase the infrastructure. Governments in Sub-Saharan African countries for infrastructure spend, on average $6-12\%$ of their gross domestic product (GDP) (UN-HABITAT (2011)).

Referring to these data, we consider the manner in which the government should provide infrastructure. Infrastructure of some kinds exists. This paper presents examination of the allocation of the infrastructure of two types: one for public investment, which increases labor productivity (transport, telecoms, and so on) and the other for health infrastructure, which raises life expectancy (water system, sanitation, hospitals, and so on).

Based on simple OLG, we discuss occupational choice, either skilled or unskilled, in addition to public investment with life expectancy. Especially, we address not only public investment but also life expectancy against dynamics. Many papers describe studies of capital accumulation by government. Barro (1990), Barro and Sala-i-Martin (1992), and Futagami, Morita and Shibata (1993) examine public capital accumulation and growth. Glomm and Ravikumar (1997) show public accumulation and growth with human capital investment. Turnovsky (1997) also discusses public capital accumulation and growth and the difference between a socially planned economy and a decentralized economy. Yakita (2008) discusses an endogenized fertility rate and an aging economy including public accumulation. Public investment in these earlier studies serves an important role in increasing productivity.

Apart from public capital accumulation, many papers have described studies of educational choice. Maoz and Moav (2000) examine skill acquisition and inequality including intergenerational mobility. Caselli (1999) and Galor and Moav (2000) introduce an idea that new technology prohibits people from learning new skills. Chen (2010) uses an overlapping generations model with life expectancy and educational choice.

Some studies have been conducted on the assumption that life expectancy is set exogenously, such as Chen (2010). However, some papers consider life expectancy as an endogenous variable. Chakraborty

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[1] As shown by UN-HABITAT (2011), in Sub-Saharan African countries, paved roads are 11.9% of all roads (2006), access to electricity is available in only 18% of households (2004), water with improved water sources is accessible to only 58% of population (2006), and only 31% of the population has access to improved sanitation facilities (2006).

(2004) and Hashimoto and Tabata (2005) set the model that life expectancy depends on public expenditure for health infrastructure, such as hospitals, clean water supply, and so on. By virtue of public expenditure, income per capita increases because capital accumulation is stimulated.[2] Chakraborty and Das (2005) and Bhattacharya and Qiao (2007) use an economy model in which private health investment can raise life expectancy and income per capita because of an increase in the saving rate.

In fact, we can consider two reasons that an economy might become enmired in a poverty trap with low income: low productivity and a low saving rate. If a government provides public infrastructure, then the productivity of labor and capital increases. Thereby, the model economy escapes from the poverty trap. However, an increase in public health infrastructure raises the saving rate and then labor productivity rises thanks to an increase in capital accumulation. Finally, the model economy escapes from the poverty trap. Our paper presents examination as the following process. First, based on Chen (2010), the paper presents public expenditure of two types (public investment and public health infrastructure) and examines what the government should provide to escape from the poverty trap. Second, after escaping from the poverty trap, the paper presents derivation of how the government allocates tax revenue between public investment and public health infrastructure to increase income growth.

As derived in this paper, the allocation for public health infrastructure to escape from the poverty trap should be within a certain range. If this allocation is large because of a decrease in public infrastructure and low productivity of labor, then the economy can not escape from the poverty trap. However, if the allocation for public investment is large, then capital accumulation is prevented and productivity of labor is low and the economy can not escape from the poverty trap. Moreover, based on the initial allocation, the government is expected to provide public infrastructure or public health infrastructure to raise the income growth rate. The results obtained in this study show how the government provides a policy to bring about income growth with a given tax revenue.

The remainder of this paper is organized as follows. Section 2 introduces the model. Section 3 presents a description of equilibrium and macroeconomic dynamics and derives the conditions under which income growth continues. Lastly, we summarize the salient points of the paper.

2   The Model

The model economy is based on a two-period (young and old) overlapping generations model. This economy has agents of three types: households, firms, and a government.

2.1   Households

Households experience two periods: young and old. During the young period, each household supplies labor inelasticity to earn labor income. This economy accommodates labor of two types: skilled labor

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[2]Hashimoto and Tabata (2005) derived the relation between health infrastructure and fertility.

and unskilled labor. Education costs must be incurred in order to become a skilled laborer, as assumed by Caselli (1999), Meckl and Zink (2004), Miyake, Muro, Nakamura, and Yasuoka (2009), and by Chen (2010). That cost is assumed as σ. Herein, w_t^s denotes the wage rate of skilled labor. The government imposes labor income taxation on the wage income of skilled labor to provide public investment and public health infrastructure.[3] Each household allocates its labor income between consumption in the young period and saving. Consequently, we obtain the following budget constraint:

$$c_{1t}^s + \frac{c_{2t+1}^s}{R_{t+1}} = (1 - \tau)w_t^s - \sigma, \tag{1}$$

$$c_{1t}^u + \frac{c_{2t+1}^u}{R_{t+1}} = w_t^u. \tag{2}$$

Indexes s and u respectively denote skilled labor and unskilled labor. In addition, c_{1t} and c_{2t+1} respectively denote consumption in the young period and old period. w_t^u denotes the wage rate of unskilled sector. R_{t+1} signifies an interest rate for annuitized savings. τ signifies labor income tax rate $(0 < \tau < 1)$. Finally, t denotes the period. A household's utility function u_t is given as shown below.

$$u_t = \alpha \ln c_{1t} + p_t(1 - \alpha) \ln c_{2t+1}, \ 0 < \alpha < 1, \ 0 < p_t < 1 \tag{3}$$

Therein, p_t denotes the probability that the individual lives during the old period. These savings are allocated among older living people if the individual dies: this is annuitized wealth. The optimal allocations at skilled labor are determined as

$$c_{1t} = \frac{\alpha}{\alpha + p_t(1 - \alpha)} \left((1 - \tau)w_t^s - \sigma \right), \tag{4}$$

$$c_{2t+1} = \frac{p_t(1 - \alpha)R_{t+1}}{\alpha + p_t(1 - \alpha)} \left((1 - \tau)w_t^s - \sigma \right). \tag{5}$$

If a worker is an unskilled laborer, then

$$c_{1t} = \frac{\alpha}{\alpha + p_t(1 - \alpha)} w_t^u, \tag{6}$$

$$c_{2t+1} = \frac{p_t(1 - \alpha)R_{t+1}}{\alpha + p_t(1 - \alpha)} w_t^u. \tag{7}$$

2.2 Firm

This paper assumes the production function shown below.[4]

$$Y_t = AK_t^\theta(G_t L_t)^{1-\theta} + B(1 - L_t), \ 0 < \theta < 1, \ 0 < A, \ 0 < B \tag{8}$$

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[3]This paper assumes that the government imposes income tax for skilled labor wage income in terms of redistribution. Assuming that the government can not capture an unskilled labor wage, the government can collect tax revenue from only skilled labor wage income.

[4]Some papers consider a production function with public investment. For example, Barro (1990) assumed $Y = K^\theta(GL)^{1-\theta}$. In addition, Caselli (1999) assumed that not only labor but also capital stock is inputted as a productive factor in the unskilled sector. Neither Caselli (1999) nor Chen (2010) considered public investment.

Therein, Y_t denotes the aggregate output. G_t and K_t respectively denote public investment and capital stock. L_t denotes the skilled labor amount. Assuming that the population size of each generation is unity, then the unskilled labor amount is shown as $1 - L_t$. With a perfectly competitive market, profit maximization reduces the following equations, as

$$w_t^s = A(1-\theta)G_t^{1-\theta}\left(\frac{K_t}{L_t}\right)^\theta,$$ (9)

$$w_t^u = B.$$ (10)

The interest rate is shown as

$$1 + r_t = A\theta\left(\frac{K_t}{G_t L_t}\right)^{\theta-1}.$$ (11)

It is noteworthy that

$$R_{t+1} = \frac{1 + r_{t+1}}{p_t}.$$ (12)

Capital stock is assumed to be fully depreciated in one period.

2.3 Government

The government imposes labor income taxation at a tax rate τ on skilled labor to provide public investment G_t and public health infrastructure H_t. Public health infrastructure is regarded as hospitals, clean water systems, and so on to raise life expectancy. Then, the government budget constraint is presented as

$$G_t + H_t = \tau w_t^s L_t.$$ (13)

Our paper assumes the following allocation rule.

$$G_t = \beta \tau w_t^s L_t$$ (14)

$$H_t = (1-\beta)\tau w_t^s L_t$$ (15)

In those equations, β $(0 < \beta < 1)$ denotes the ratio of public investment to tax revenue and $1 - \beta$ denotes that of public health infrastructure. Moreover, this paper includes the assumption of life expectancy as $p_t = \min[CH_t^\epsilon, 1]$ $(C > 0$ and $0 < p_t < 1)$.

3 Equilibrium

This section presents derivation of the equilibrium of this model economy. If workers move freely between two sectors, then the indifference condition is described as

$$w_t^s = \frac{B + \sigma}{1 - \tau} \equiv \hat{w}.$$ (16)

Considering (9), (14), and (16), L_t is given as

$$L_t = \min\left[\left(\frac{(1-\tau)\tau^{\frac{1-\theta}{\theta}}(1-\theta)^{\frac{1}{\theta}}A^{\frac{1}{\theta}}\beta^{\frac{1-\theta}{\theta}}K_t}{B+\sigma}\right)^{\frac{\theta}{2\theta-1}},1\right]. \tag{17}$$

Intuitively, an increase in K_t raises L_t because an increase in w_t^s brings about an increase in the amount of skilled labor. Therefore, we assume that $\theta > \frac{1}{2}$.[5] Additionally, we assume the total population size in each generation as unity. Then, the dynamics of capital stock at $L_t < 1$ is derived as shown below:

$$K_{t+1} = \frac{p_t(1-\alpha)}{\alpha + p_t(1-\alpha)}\left(((1-\tau)w_t^s - \sigma)L_t + B(1-L_t)\right) = \frac{p_t(1-\alpha)B}{\alpha + p_t(1-\alpha)}, \tag{18}$$

where $p_t = C\left(\frac{\tau(1-\beta)(B+\sigma)L_t}{1-\tau}\right)^{\epsilon}$.[6] This paper assumes $C\left(\frac{\tau(B+\sigma)}{1-\tau}\right)^{\epsilon} < 1$. In contrast, an increase in K_t raises L_t. Therefore, the dynamics of capital stock at $L_t^s = 1$ is derived as

$$K_{t+1} = \frac{p_t(1-\alpha)}{\alpha + p_t(1-\alpha)}\left(XK_t - \sigma\right), \tag{19}$$

where $p_t = \min\left[C\left((1-\beta)\tau^{\frac{1}{\theta}}A^{\frac{1}{\theta}}(1-\theta)^{\frac{1}{\theta}}\beta^{\frac{1-\theta}{\theta}}K_t\right)^{\epsilon},1\right]$ and $X = (1-\tau)A^{\frac{1}{\theta}}(1-\theta)^{\frac{1}{\theta}}\beta^{\frac{1-\theta}{\theta}}\tau^{\frac{1-\theta}{\theta}}$.[7] Then, w_t^s and $1 + r_t$ at $L_t = 1$ are

$$w_t^s = \beta^{\frac{1-\theta}{\theta}}\tau^{\frac{1-\theta}{\theta}}(1-\theta)^{\frac{1}{\theta}}A^{\frac{1}{\theta}}K_t, \tag{20}$$

$$1 + r_t = \beta^{\frac{1-\theta}{\theta}}\tau^{\frac{1-\theta}{\theta}}\theta(1-\theta)^{\frac{1-\theta}{\theta}}A^{\frac{1}{\theta}}. \tag{21}$$

Therefore, calculating $Y_t = (1 + r_t)K_t + w_t^s L_t$, we obtain $Y_t = \beta^{\frac{1-\theta}{\theta}}\tau^{\frac{1-\theta}{\theta}}(1-\theta)^{\frac{1-\theta}{\theta}}A^{\frac{1}{\theta}}K_t$. The level of capital stock $\hat{K}$ that brings about $w_t^s = \hat{w}$ and $L_t = 1$ is

$$\hat{K} = \frac{B+\sigma}{(1-\tau)\tau^{\frac{1-\theta}{\theta}}\beta^{\frac{1-\theta}{\theta}}(1-\theta)^{\frac{1}{\theta}}A^{\frac{1}{\theta}}}. \tag{22}$$

Then, we can depict the dynamics of K_t as presented below.

[Insert Fig. 1 around here.]

Assuming $(1-\alpha)(1-\tau)A^{\frac{1}{\theta}}(1-\theta)^{\frac{1}{\theta}}\beta^{\frac{1-\theta}{\theta}}\tau^{\frac{1-\theta}{\theta}} > 1$, there exist dynamics of two types. The first is that income growth occurs for any K_0 (Fig. 1-1). The second is that income growth occurs or does not occur for given K_0 (Fig. 1-2). In Fig. 1-1, given an initial K_0, income growth continues. Then, only skilled labor exists. However, in Fig. 1-2, given K_0 less than $\tilde{K}$, the capital stock converges to K^*, which exists for both unskilled labor and skilled labor. It is the poverty trap bringing about low income B. The condition not to have the poverty trap is $\frac{p_t(1-\alpha)B}{\alpha+p_t(1-\alpha)} > \frac{B+\sigma}{(1-\tau)\tau^{\frac{1-\theta}{\theta}}\beta^{\frac{1-\theta}{\theta}}(1-\theta)^{\frac{1}{\theta}}A^{\frac{1}{\theta}}}$; that is,

$$1 > \frac{B+\sigma}{(1-\alpha)B(1-\tau)\tau^{\frac{1-\theta}{\theta}}(1-\theta)^{\frac{1}{\theta}}A^{\frac{1}{\theta}}}\frac{\alpha + p_t(1-\alpha)}{p_t\beta^{\frac{1-\theta}{\theta}}}, \tag{23}$$

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[5] $1 - \theta$ denotes the elasticity of output Y for public investment G, i.e., $\frac{dY}{dG}\frac{G}{Y}$. Our paper assumes small elasticity such as Barro (1990). Barro (1990) assumed $1 - \theta = 0.25$ and simulated the model economy.

[6] Considering (17) and $p_t = C\left(\frac{\tau(1-\beta)(B+\sigma)L_t}{1-\tau}\right)^{\epsilon}$, we obtain $\frac{\partial K_{t+1}}{\partial K_t} > 0$ and $\frac{\partial^2 K_{t+1}}{\partial K_t^2} < 0$.

[7] See Appendix for the form of this dynamics equation.

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where $p_t = C\left(\frac{\tau(1-\beta)(B+\sigma)}{1-\tau}\right)^{\epsilon}$. Defining the right-hand-side of this inequality as $F(\beta)$, we obtain $\lim_{\beta \to 0} F(\beta) = \infty$ and $\lim_{\beta \to 1} F(\beta) = \infty$. In addition, β^* exists to hold $\frac{\alpha \epsilon \theta}{1-\theta}\frac{\beta}{1-\beta} = (1-\alpha)p_t + \alpha$. With $\beta < \beta^*$, the sign of $\frac{dF(\beta)}{d\beta}$ is negative and the sign is positive if $\beta > \beta^*$. Then the following figure is shown.

[Insert Fig. 2 around here.]

The solid line is given by the following condition as

$$1 > \frac{B+\sigma}{(1-\alpha)B(1-\tau)\tau^{\frac{1-\theta}{\theta}}(1-\theta)^{\frac{1}{\theta}}A^{\frac{1}{\theta}}}\frac{\alpha+p_t(1-\alpha)}{p_t\beta^{*\frac{1-\theta}{\theta}}}, \tag{24}$$

where $p_t = C\left(\frac{\tau(1-\beta^*)(B+\sigma)}{1-\tau}\right)^{\epsilon}$. Otherwise, the dashed line is given. At the solid line, if β exists between β_0 and β_1, then the dynamics is shown in Fig. 1-1 and the poverty trap does not exist. Therefore, even if the economy stays in the poverty trap shown by K^*, the economy escapes from the poverty trap and income growth continues as the government sets β within $\beta_0 < \beta < \beta_1$. However, with the parametric condition to hold the dashed line, the economy can not escape from the poverty trap. Then, the following proposition is established.

Proposition 1 If Eq. (24) holds, then the economy can escape from the poverty trap and income growth continues to set β within $\beta_0 < \beta < \beta_1$.

This proposition is intuitive. If β is small, then public health infrastructure is large but public investment is small. Small public investment decreases labor productivity and the income level. Then, the saving is small and capital accumulation is not large. Therefore, the economy can not escape from the poverty trap. However, if β is large, then public investment is large but public health infrastructure is small. Small public health infrastructure brings about short life expectancy and the saving rate is low.

However, if the condition of (24) does not hold, then the government can not induce the economy to escape from the poverty trap with the allocation of β. The government must collect more tax revenue and allocate public investment and public health infrastructure. However, an increase in tax burden decreases capital accumulation because of a decrease of the saving. If the following condition

$$\left(\frac{1}{1-\tau} - \frac{1-\theta}{\theta}\right)(\alpha + (1-\alpha)p_t) + \frac{\alpha\epsilon}{1-\beta} < 0 \tag{25}$$

is held, then an increase in τ shifts down the curve of $F(\beta)$.[8] Defining τ^* to equalize (25), $F(\beta^*)$ at $\tau = \tau^*$ is larger than one, then the economy can not escape from the poverty trap even if the government changes τ and β. The following proposition is established.

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[8] We obtain this condition as $\frac{dF(\beta)}{d\beta} < 0$. This condition shows that the effect to decrease capital accumulation by an increase in τ is small.

Proposition 2 If $F(\beta^*)$ at $\tau = \tau^*$ is larger than one, then the economy always stays in the poverty trap even if the government changes τ and β.

Next, we consider that the economy escapes from the poverty trap and that income growth continues and examine how the allocations of β affect income growth. Calculating $\frac{dK_{t+1}}{d\beta}$ at (19) in $p_t < 1$, we obtain the following equation.

$$\frac{dK_{t+1}}{d\beta} = \frac{(1-\alpha)p_t}{\beta\theta(\alpha+(1-\alpha)p_t)}\left((1-\theta)XK_t + \frac{\alpha\epsilon(1-\beta-\theta)}{(\alpha+(1-\alpha)p_t)(1-\beta)}(XK_t - \sigma)\right). \tag{26}$$

If $1 - \beta - \theta > 0$, then the sign of $\frac{dK_{t+1}}{d\beta}$ is positive. In $1 - \beta - \theta < 0$, if

$$p_t < \frac{\alpha\epsilon(\beta+\theta-1)}{(1-\alpha)(1-\beta)(1-\theta)} - \frac{\alpha}{1-\alpha}, \tag{27}$$

$$K_t < \frac{\alpha\epsilon(1-\theta-\beta)\sigma}{((1-\beta)(1-\theta)(\alpha+(1-\alpha)p_t)+\alpha\epsilon(1-\theta-\beta))X}, \tag{28}$$

the sign of $\frac{dK_{t+1}}{d\beta}$ is positive. Then the following proposition is established.

Proposition 3 If income growth continues and $1 - \beta - \theta > 0$, then an increase in β can always raise the income growth rate. If $1 - \beta - \theta < 0$ and the condition given by Eqs. (27) and (28) holds, then an increase in β can raise the income growth rate.

An increase in β increases wage rate w_t^s and the saving increases, too. However, an increase in β reduces public health infrastructure. This effect decreases capital accumulation. Moreover, an increase in β raises tax revenue because of an increase in public investment and reduces tax revenue because of a decrease in capital accumulation because of a decrease in life expectancy. If $1 - \beta - \theta > 0$, then the effect of an increase in β raises tax revenue dominates; then life expectancy rises. However, if $1 - \beta - \theta < 0$, then the effect of an increase in β decreases life expectancy predominantly. If this effect is large, then an increase in β reduces income growth.

The income growth rate in this model economy converges to $\frac{K_{t+1}}{K_t} = X$. We note that $\lim_{K_t \to \infty} \frac{\sigma}{K_t} = 0$ and $\lim_{K_t \to \infty} p_t = 1$. Then, an increase in β can always raise income growth because an increase in β does not affect life expectancy, which is a sufficiently large level.

4 Concluding and Remarks

This paper described a model with public investment and public health infrastructure and illustrated how an increase in public investment and public health infrastructure affect capital stock, the amount of skilled labor, and the wage rate. First, the government must adequately allocate public investment and public health infrastructure to escape from the poverty trap. Public investment and an increase

in life expectancy with public health infrastructure can stimulate capital accumulation. Therefore, the government sets these allocations not to be disturbed these both effects. Second, if the economy can escape from the poverty trap and income growth continues, then an increase in public investment can not always raise income growth. However, if the allocation of tax revenue for public health infrastructure is large or capital accumulation is small, an increase in public investment can raise income growth.

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Appendix

Form of the dynamics equation

The sign of $\frac{dK_{t+1}}{dK_t}$ of (19) is positive. The sign of $\frac{\partial^2 K_{t+1}}{\partial K_t^2}$ is calculated as follows.

$$\frac{\partial K_{t+1}}{\partial K_t^2} = \frac{2\alpha(1-\alpha)p_t'\frac{\partial H_t}{\partial K_t}X}{(\alpha+(1-\alpha)p_t)^2}$$

$$+ \frac{\left(p_t''\frac{\partial H}{\partial K} + p_t'\frac{\partial^2 H_t}{\partial K_t^2}\right)(\alpha+(1-\alpha)p_t) - 2(1-\alpha)p'^2\left(\frac{\partial H_t}{\partial K_t}\right)^2}{(\alpha+(1-\alpha)p_t)^4}\alpha(1-\alpha)(\alpha+(1-\alpha)p_t)(XK_t - \sigma),$$

where $p' = \frac{\partial p}{\partial H}$, $H_t = \left((1-\beta)\tau^{\frac{1}{\theta}}\beta^{\frac{1-\theta}{\theta}}(1-\theta)^{\frac{1}{\theta}}A^{\frac{1}{\theta}}K_t\right)^\epsilon$. The first term of the right hand in this equation is positive. The sign is ambiguous because the second term is negative. However, an increase in K_t the slope $\frac{(1-\alpha)p_t}{\alpha+(1-\alpha)p_t}$ from zero to $(1-\alpha)$. We find that $(XK_t - \sigma)$ increases with K_t. Therefore, $\frac{(1-\alpha)p_t}{\alpha+(1-\alpha)p_t} \times (XK_t - \sigma)$ increases with K_t and the amount of increase raises with K_t. Consequently, $\frac{dK_{t+1}}{dK_t} > 0$ and $\frac{\partial^2 K_{t+1}}{\partial K_t^2} > 0$.
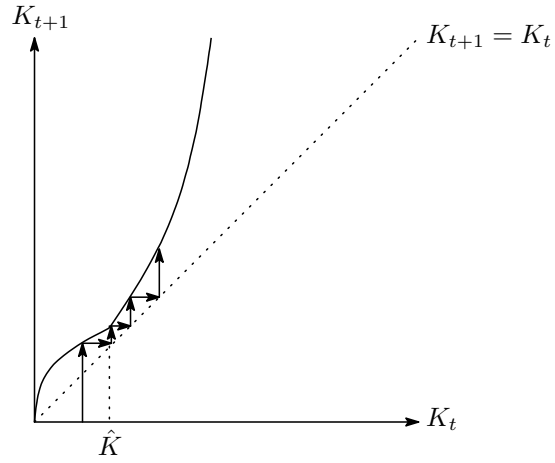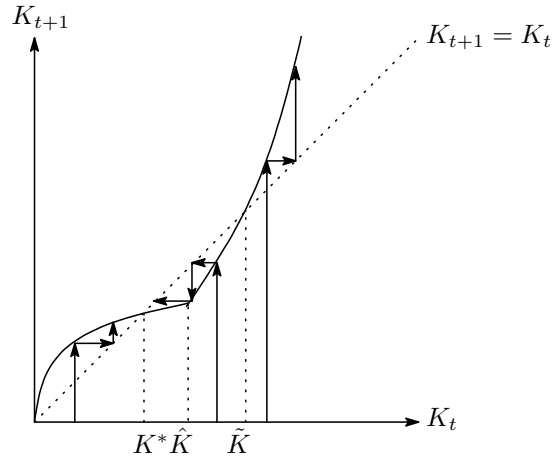
Fig. 1-1: Dynamics of K_t (Income growth).

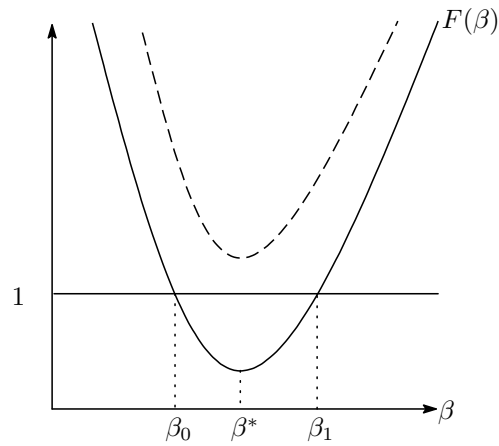Fig. 1-2: Dynamics of K_t (Income growth or no income growth).

Fig. 2: Range of β not to stay in the poverty trap.